Intro to Formal Political Analysis:
Why Game Theory? What is Game Theory?

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- Formal language for strategic interactions
- Robust predictions about human behavior?
- **Caveat!** Like all models, only as good as the appropriateness of the assumptions

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- $\succsim$ is a preference relation over outcomes.

Preferences

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- **Note** that this does **not** mean the decision maker's preferences have to be qualitatively "rational" or "self-interested"

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  - A payoff function *v* where *v*(orange) = 1, *v*(apple) = 10, and *v*(banana) = 100 would also represent Pat's preferences